Assessing Damages in Antitrust Litigation

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Abstract

Victims of antitrust violations can recover damages in court. Yet, the quantification of antitrust damages and to whom they accrue is often complex. An illegal price increase somewhere in the chain of production percolates through to the other layers in a ripple of partial pass-ons. The resulting reductions in sales and input demands lead to additional harm to both downstream (in)direct purchasers and upstream suppliers. Nevertheless, US civil antitrust litigation is almost exclusively concerned with direct purchaser claims for (treble) damages calculated on the basis of the overcharge. In this paper, we show that there is no structural relationship between the direct purchaser overcharge and the true harm inflicted by an antitrust violation on all direct and indirect purchasers and sellers in the chain of production.

1 Introduction

Anticompetitive acts to eliminate competition and prevent new entry can cause severe and widespread harm. In the US, under Section 4 of the Clayton Act: "Any person who will be injured in his business or property by reason of anything forbidden in the antitrust laws [...] shall recover threefold the damages by him sustained." The vast majority of civil actions for antitrust damages concerns cartels.

The identification of antitrust harm can be complicated. In longer supply chains, in which one product is an input in the production of the next, an illegal price increase somewhere in the chain can percolate through to the other layers in a ripple of pass-ons. The resulting reductions in sales cause additional harm to direct and indirect customers and suppliers of the wrongdoer(s).

In order to determine who is harmed by an antitrust violation and to what extent, in principle all actual trades need to be compared to what would have been the market allocation without the anticompetitive behavior – the so-called "but-for" world [1] In practice this is often difficult. At a minimum, it requires information about consumer demand and the structure of the market, such as the number of layers in the production chain, the type

[1] See Fischer (2006) for a survey of some of the methods that can be applied in the determination of but-for prices

and level of competition amongst firms in each layer, their production technologies, and costs.

In the US, some of these complexities have been reduced by case law. At least since *Chattanooga Foundry* (1906) have direct purchasers been entitled to recover damages on the basis of the overcharge they paid as a result of antitrust infringement.[2] According with this prevailing method, basic damages – before trebling and interest, if applicable – are calculated as the difference between the anticompetitive price and the competitive but-for price multiplied by the amount actually purchased. The overcharge ignores lost profits on transactions that could have been made at lower prices, which courts have been reluctant to award.

Indirect purchasers often do not have standing to sue. In *Hanover Shoe* (1968), the Supreme Court ruled against the use of the pass-on defense in Federal antitrust damage actions.[3] In a pass-on defense, the defendant attempts to show that the plaintiff did not in fact suffer the amount of damages claimed on the argument that it was not able to pass on all of the overcharges on downstream to its customers. In addition, in *Illinois Brick* (1977) the Supreme Court established that only the direct purchasers have legal standing in Federal court to sue for antitrust damages.[4] Later, in *California v. ARC America Corp.* 490 US 93 (1989) the Supreme Court left it to the discretion of individual states whether or not to allow indirect purchaser suits. As a result, the rules on antitrust standing vary across the states, with presently a small majority allowing indirect purchaser suits under state law. *Hanover Shoe* and *Illinois Brick* together cemented the use of the overcharge, which indeed disregards pass through.

Direct suppliers to a buyers' cartel that colluded to depress input prices can in principle maintain a treble-damages action after *Mandeville Island Farms* (1948).[5]Yet, Standing was denied by the Supreme Court to suppliers damaged by anticompetitively restricted demand for their produce in *Associated Contractors* (1983).[6] Suppliers' standing is discussed more extensively in Section 4. As a result of these legal constraints, in the vast majority of US antitrust damages actions, the plaintiffs are direct purchasers and their claim is based on the overcharge. In this paper, we consider the effects of anticompetitively raised prices somewhere in a chain of production with an arbitrary number of layers. We will mostly refer to cartels, but our results have wider antitrust application. Competition in each layer is specified between perfect competition and monopoly. This allows us to exactly characterize the effects of the cartel's direct and indirect purchasers, as well as its direct and indirect suppliers. We assess the bias introduced by relying on the overcharge on the direct purchasers – which we refer to as the "direct purchaser overcharge" – for the estimation of the actual antitrust harm in the chain.

[2] *Chattanooga Foundry Pipe Works v. Atlanta*, 203 US 396 (1906).
[3] *Hanover Shoe, Inc. v. United Machinery Corp.*392 US 481 (1968)
[4] *Illinois Brick Co. v. Illinois* US 720 (1977)
[5] *Mandeville Islands Farms v. American Crystal Sugar Co.*, 334 US 219 (1948).
[6] *Associated Ge, Contractors of Cal., Inc. v. Carpenters*, 459 US 519 (1983).

We find that even in the most basic of settings – with unit pricing and input taking – the direct purchaser overcharge is a poor measure of the true antitrust harm. The overcharge can grossly underestimate the actual antitrust harm depending on such characteristics as the market shape of demand, the number of producers, the type of competition, and the location of the cartel in the chain of production. In particular, we show that lost profit harm ignored by the direct purchaser overcharge may increase without bound with the length of the production chain. Moreover, the method misses harm sustained upstream from the cartel, which can be substantial. The ratio of antitrust harm to the direct purchaser overcharge can be anything between one and infinity.

The existing literature on cartel pass-on effects uses a model with only three layers: a top layer of input producers that form a cartel upstream, a layer of direct purchasers downstream who sell to a third layer of final consumers.[7] Hellwig (2006) shows that the deadweight-loss of a direct purchaser overcharge is a good measure for the actual antitrust harm sustained by this group. Verboven and van Dijk (2007) use the mainstream model to analyze an infinitesimal cartel price increase to determine "discounts" to be given on the direct purchaser overcharge to correct for pass-ons to consumers and output effects locally. Basso and Ross (2007) extend the approach to differentiated products, so that there can be input substitution, to produce a numerical table of correction factors for a discrete cartel price increase. For all practical purposes, Boone and Muller (2008) express the share of (otherwise unspecified) total antitrust harm borne by consumers for an infinitesimal price increase as a function of common measures such as HHI and PCM.

This paper is organized as follows. In Section 2, we decompose the various welfare effects caused by a cartel anywhere in a chain of production and relate aggregate and individual effects to the overcharge on direct purchasers. In Section 3, we evaluate the direct purchaser overcharge as an estimator of antitrust harm. Section 4 concludes. Appendix A illustrates our decomposition of harm. Appendix B presents an example of an upstream "undercharge" without collusive power. Appendix C contains the proofs.

2 Cartel Effects in a Chain of Production

2.1 A Vertical Model of Production

Consider a vertical chain with several layers of intermediaries, each adding value to produce a homogeneous consumer product. Let consumer demand for the final product be represented by an inverse demand function $P : \mathbb{R}_+ \rightarrow \mathbb{R}_+$, which is nonincreasing, twice differentiable and contiguous in aggregate production Q. Let there be K layers of production, with n_k firms active in layer k. Except for layer 1, where the raw materials originate, the firms in any layer k each transform a homogeneous input they purchase from firms in layer $k-1$, using a one-to-one technology, into a homogeneous new output, which they sell

[7] See Harris and Sullivan (1979) and Kosicki and Cahill (2006).

on to the firms in layer $k+1$. Eventually, the firms in layer K sell the final product to consumers. Figure 1 illustrates.

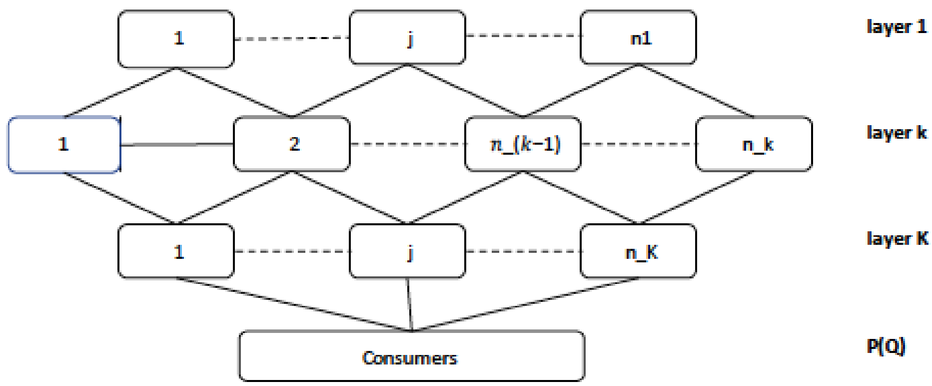

Figure 1: A Longer Vertical Chain of Production

We assume that the number of firms in each layer is exogenously given and fixed. The cost function for firm j in layer k is given by $p_{k-1}q + c_{jk}(q)$, where p_{k-1} is the unit price for the input from layer $k-1$ (with $p_0 = 0$), and $c_{jk}(q)$ are the costs for transforming q units of the input into q units of the output. We abstract from nonlinear pricing or more general types of vertical relations between firms from different layers.

Firms move simultaneously within the same horizontal layer, and sequentially following the layer above – so layer 1 moves first, layer 2 second, and so on. That is, we assume that all firms in each layer purchase from the producers in the layer above at going prices, without bargaining. As a result, all upstream antitrust effects in our model are reduced input demand effects.

The market equilibrium is found by backward induction. First consider layer K. For any possible input price p_{K-1} – and given final consumer demand $P(Q)$ – we can determine the resulting equilibrium output in layer K. Let the relationship between this equilibrium quantity and p_{K-1} be represented by a uniquely defined, nonincreasing, continuous, and differentiable function $p_{K-1}(Q)$. This function serves as the inverse demand function for the firms in layer $K-1$.[8] Firms in layer $K-1$ then determine, for any p_{K-2} – and given their inverse demand function $p_{K-1}(Q)$ – their optimal production quantity, in turn leading to an inverse demand function $p_{K-2}(Q)$ for firms in layer $K-2$, and so on.

For analytical convenience, we analyze a model with conjectural variations to simulate

[8]This is an assumption insofar that *a priori* it can be that for a certain value of p_k there exist multiple equilibria in the quantity-setting subgame in layer k. For all specifications considered in this paper, however, an explicit, continuous, and differentiable relationship between p_k and Q exists for every k.

various types of competition in each layer.[9] That is, in each layer k firm j's conjecture about the reaction of the other firms in that layer to its quantity decision is $\vartheta_k = \frac{\partial Q}{\partial q_{jk}}$. We assume that ϑ_k is the same for all firms in a horizontal layer, but may be different for firms from different layers vertically. Hence, given the input price p_{k-1}, the first-order condition for a symmetric equilibrium in layer k, with conjectural variations parameter ϑ_k is

$$p_k(Q) + \frac{\vartheta_k}{n_k} Q p_k'(Q) - c_k - p_{k-1} = 0 \tag{1}$$

Note that the classic Cournot conjecture corresponds to $\vartheta_k = 1$. If $\vartheta_k = 0$, all firms in layer k are price takers, so that the equilibrium prices will equal marginal costs, *i.e.*, $p_k = p_{k-1} + c_k$. The specification $\vartheta_k = n_k$ is analytically equivalent to full horizontal collusion in layer k. Other values of ϑ_k close to n_k can be interpreted as forms of imperfect collusion, in which joint-profit maximization is further constrained, for example when the cartel members understand that the risk of discovery and the size of a consequential damage claim are likely to depend upon the cartel's pricing and production strategy.[10]

We denote the ultimate equilibrium quantity on the inverse demand function $p_1(Q)$ faced by the firms in layer 1 by Q^*. Equilibrium prices then clear as $p_1^* = p_1(Q^*), ..., p_{K-1}^* = p_{K-1}(Q^*)$ and $p_K^* = P^* = P(Q^*)$. The individual level of production of firm j in layer k equals q_{jk}^*, with $\sum_{j=1}^{n_k} q_{jk}^* = Q^*$. Equilibrium profits and consumer surplus follow straightforwardly from these quantities and prices.

Now suppose that the firms in some layer $g \in 1, ..., K$ form a cartel, while competitive conditions in all other layers remain the same – note that these may include pre-existing cartels elsewhere in the chain. Our setup implies that the cartel uses its obtained market power to raise unit prices *vis-a-vis* its customers, but remains a price-taker on the market for its inputs. We further assume that there are no cartel-specific efficiency gains that would somehow allow the cartel to produce at lower costs than its members could in competition. Let the resulting equilibrium quantity and equilibrium prices under the cartel regime be denoted by Q^g and p_k^g for $k = 1, ..., K$. [11] Firm j in layer k produces q_{jk}^g with $\sum_{j=1}^{n_k} q_{jk}^g = Q^g$, for all k.

2.2 Decomposition of Cartel Effects

The presence of the cartel causes harm to welfare in the form of high unit prices, resulting in lost profits throughout the chain of production, lost consumers' surplus, and deadweight-

[9] Basso and Ross (2007) take the same approach. For a conceptual critique of conjectural variations, see Hahn (1989).

[10] In our model it is optimal for the cartel to increase prices symmetrically, so that the cartel price p_g^q is the same for each direct purchaser. Verboven and van Dijk (2007) also consider various asymmetric cartel price mark-ups in their reduced-form model. This could be relevant for exampleif some of the direct purchasers are integrated with a colluding firm, however Verboven and van Dijk (2007) do not derive mark-up differentiation as an optimal pricing strategy.

[11] For an analysis of comparative static effects in Cournot models, see Dixit (1986) and Quirmbach (1988)

losses, while the cartel members raise their profits. That is, $Q^g < Q^*$ and $p_g^g < p_g^*$. Typically also $p_k^g > p_k^*$ for $k > g$ and downstream intermediaries and consumers are harmed by the price conspiracy. Under certain specifications, the profits of some downstream intermediaries – in particular direct purchasers – may actually increase in response to the upstream price increases. Also, prices higher up in the chain may either increase or decrease, depending on the shape of demand and cost functions. *In toto*, however, collusion on unit prices is always bad for welfare.

The impact of g-level cartel's unit price increase on one particular layer k of production that is downstream from layer g can be decomposed into three distinct effects. The *overcharge effect* on layer k is the amount by which the firms in this layer are overcharged by the previous layer $k-1$. Part of the burden of this overcharge may be passed on by firms in layer k to the next layer of production, layer $k+1$. This is the *pass-on effect.* Finally, the *output effect* results from the decrease in production due to the cartel: it amounts to the losses in profits from the reduction in sales.[12] We consider each of these effects separately, as they are borne out in lost profits.

Consider the aggregate profits of firms in layer k. In the competitive benchmark, these are $\pi_k^* = (p_k^* - p_{k-1}^*)Q^* - \sum_{j=1}^{n_k} c_{jk}(q_{jk}^*)$. Under the cartel regime, they are $\pi_k^g = (p_k^g - p_{k-1}^g)Q^g - \sum_{j=1}^{n_k} c_{jk}(q_{jk}^g)$. The difference $\Delta\pi_k = \pi_k^* - \pi_k^g$ can be decomposed as follows:

$$\Delta\pi_k = Q^g(p_{k-1}^g - p_{k-1}^*) - Q^g(p_k^g - p_k^*) + \left[(Q^* - Q^g)(p_k^* - p_{k-1}^*) + \sum_{j=1}^{n_k} c_{jk}(q_{jk}^g - \sum_{j=1}^{n_k} c_{jk}(q_{jk}^*)\right] = \zeta_k - \omega_k + \sigma_k \tag{2}$$

The first factor is the overcharge effect on firms in layer k, thus defined as:

$$\zeta_k = Q^g(p_{k-1}^g - p_{k-1}^*) \tag{3}$$

or the price increase of the product of the previous layer $k-1$, multiplied by the quantity purchased under the cartel regime.

The second factor:

$$\omega_k = Q^g(p_k^g - p_k^*) \tag{4}$$

corresponds to the pass-on effect, which is the amount of the price increase that layer k passes on to layer $k+1$. It is equal to the price increase of layer k multiplied by the quantity produced under the cartel regime. Note that ω_k, the pass-on effect of layer k, equals the overcharge effect suffered by layer $k+1$, that is $\zeta_{k+1} = \omega_k$.

[12] Our decomposition follows Hellwig (2006), but we use a slightly different nomenclature. Where Hellwig distinguishes between a *direct cost effect* and a *business loss effect*, we use *overcharge effect* and *output effect*, respectively. Verboven and van Dijk (2007) also analyze both effects, but for exogenous infinitesimal changes in the input prices, rather than the endogenous discrete equilibrium effects. They speak of *direct cost effect* and *output effect*, respectively. All three papers share the definition of the *pass-on effect.*

The last factor in equation (2) is the output effect:

$$\sigma_k = (Q^* - Q^g)(p_k^* - p_{k-1}^*) + \sum_{j=1}^{n_k}(c_{jk}(q_{jk}^g) - c_{jk}(q_{jk}^*)) \quad (5)$$

This part represents the loss of profits that could have been made on the larger volume in the competitive benchmark. It can be rewritten as the sum of individual firm output effects, $\sigma_k = \sum_{j=1}^{k} \sigma_{jk}$, with

$$\sigma_{jk} = (q_{jk}^* - q_{jk}^g)(p_k^* - p_{k-1}^* - \bar{c_{jk}}(q_{jk}^*)) + q_{jk}^g(\bar{c_{jk}}(q_{jk}^g) - \bar{c_{jk}}(q_{jk}^*))$$

. Here, $\bar{c_{jk}}(q_{jk} = c_{jk}(q_{jk})/q_{jk}$ are the average costs from firm j in layer k, evaluated at q_{jk}. The first part of the individual output effect, $(q_{jk}^* - q_{jk}^g)(p_k^* - p_{k-1}^* - \bar{c_{jk}}(q_{jk}^*))$, equals the lost sales times the average profit margin and is always positive. The sign of the second part, $q_{jk}^g(\bar{c_{jk}}(q_{jk}^g) - c_{jk}\bar{(}q_jk^*))$, is ambiguous. It is positive (negative) if the average costs for firm j in layer k are decreasing (increasing).

The effect on layers *upstream* from cartel layer g – *i.e.*, layers $k < g$ – can be decomposed in much the same way. These layers also each face an overcharge, a pass-on, and an output effect. The effect of the cartel on upstream prices result from reduced derived demand and are ambiguous. As a result, so are the signs of the upstream overcharge and pass-on effects. If all upstream prices increase, the upstream overcharge and pass-on effects are positive. If all upstream prices decrease, both the overcharge and pass-n effects on the upstream layers will be negative, corresponding to a decrease in input costs and a decrease in revenues, respectively. In certain specifications, it may also be that all upstream prices remain the same so that there are no upstream overcharge and pass-on effects. Generally, some of the upstream prices may increase and others decrease.

All the way at the end of the supply chain, the loss in consumer surplus of the final consumer is given by:

$$\Delta CS = CS^* - CS^g = \zeta_C + \sigma_C$$

with

$$\zeta_C = Q^g(p_K^g - p_K^*) and. \sigma_C = \int_{Q^g}^{Q^*} [P(Q) - P(Q^*)]dQ$$

where $p_K^g = P(Q^g)$ and $p_K^* = P(Q^*)$. Note that, since these are the final consumers, there is no pass-on effect. Also note that, because $Q^g < Q^*$, both ζ_C and σ_C are strictly positive and final consumers unambiguously suffer from the cartel.

Our decomposition of cartel effects straightforwardly allows us to formulate the following basic insight;

Proposition 1 *The direct purchaser overcharge is equal to the sum of all downstream overcharges, net of pass-ons,* $\sum_{k=1}^{K}(\zeta_k - \omega_k) + \zeta_C = \zeta_{g+1}$.

Note that in an overcharge conception of compensatory damages, this result justifies obtaining the direct purchaser overcharge proceeds first, and then redistributing them amongst indirect purchasers later.

2.3 Measure of Antitrust Harm

We are primarily interested in the relationship between the direct purchaser overcharge and the net actual antitrust harm to total welfare. The latter is equal to the change in total profits in the chain, $\sum_{k=1}^{K} \Delta\pi_k$, plus the change in consumer surplus, ΔCS. That is,

$$\Delta W = \sum_{k=1}^{g-1} \Delta\pi_k + \Delta\pi_g + \left[\sum_{k=g+1}^{k} \Delta\pi_k + \Delta CS \right] = d_U + \Delta\pi_g + d_D$$

. The *cartel gains* are $\Delta\pi_g$. The *downstream damages* $d_D = \sum_{k=g+1}^{K} \Delta\pi_k + \Delta CS$ correspond to losses in profits and consumer surplus by all direct and indirect purchasers. In addition, there are *upstream damages*, $d_U = \sum_{k=1}^{g-1} \Delta\pi_k$, equal to profit losses incurred by direct and indirect suppliers to the cartel.

We can use our decomposition of harm in equation (2) to evaluate each of these terms separately. We find:

$$\Delta W = \sum_{k=1}^{K} (\zeta_k - \omega_k + \sigma_k) + \zeta_C + \sigma_C = \sum_{k=1}^{K} \sigma_k + \sigma_C$$

, where we used $\zeta_1 = 0$ and the fact that the overcharge on layer $k+1$ equals the pass-on of layer k, $\zeta_{k+1} = \omega_k$ for $k = 1, ..., K$ and $\zeta_C = \omega_K$. The total welfare effect therefore coincides with the sum of the output effects.

Cartel profits are $\Delta\pi_g = \zeta_g - \omega_g + \sigma_g$. Downstream harm can be represented as:

$$d_D = \sum_{k=g+1}^{K} (\zeta_k = \omega_k + \sigma_k) + \zeta_C + \sigma_C = \zeta_{g+1} + \sum_{k=g+1}^{K} \sigma_k + \sigma_C$$

, or the sum of all output effects of direct and indirect purchasers plus the direct purchaser overcharge. Upstream harm is equal to:

$$d_U = \sum_{k=1}^{g-1} (\zeta_k - \omega_k + \sigma_k) = -\omega_{g-1} + \sum_{k=1}^{g-1} \sigma_k = -\zeta_g + \sum_{k=1}^{g-1} \sigma_k$$

.

We study the direct purchaser overcharge, $\zeta g + 1$, in relation to these actual welfare effects. That is, we evaluate the ratio:

$$\lambda_W = \frac{\Delta W}{\zeta_{g+1}} = \lambda_g + \lambda_D + \lambda_U$$

in which:

$$\lambda_g = \frac{\Delta\pi_g}{\zeta_{g+1}} = \frac{\zeta_g + \sigma_g}{\omega_g} - 1 \tag{6}$$

are the cartel gins expressed in the direct purchaser overcharge,

$$\lambda_D = \frac{d_D}{\zeta_{g+1}} = 1 + \sum_{k=g+1}^{K}$$

In addition to these aggregate measures, we specify the individual harm to direct purchasers and consumers as:

$$\lambda_{g+1} = \frac{\zeta_{g+1} - \omega_{g+1} + \sigma_{g+1}}{\zeta_{g+1}} = 1 - \frac{\omega_{g+1} - \sigma_{g+1}}{\zeta_{g+1}} \quad \text{and} \quad \lambda_c = \frac{\zeta_C + \sigma_C}{\zeta_{g+1}} \tag{8}$$

In the next section, we evaluate how these various ratios vary with the intensity of competition, the number of firms in each layer, the number of layers, and the position of the layer in which the uncompetitive behavior emerges in a specified vertical production model.

3 Quantifying Antitrust Damages Using the Direct Purchaser Overcharge

In order to explicitly characterize the ratios introduced above, we need to further specify our model. Suppose that marginal costs are constant and identical for every firm in the same layer. That is, for layer$k$ we have $c_{jk}q = c_kq$, for each $j \in 1, ..., n_k$[13] Let the inverse demand function be:

$$P(Q) = a - bQ^\gamma \tag{9}$$

with a, b, and $\gamma > 0$.[14] Inverse demand is a convex function of quantity for $0 < \gamma < 1!, a concave function for \gamma > 1$, and a linear function for $\gamma = 1$.

In this setup, the equilibrium quantity and prices cam be expressed as follows:

$$Q^* = \left[\frac{1}{b}\left(\prod_{i=1}^{K} \frac{n_i}{n_i + \gamma^{\vartheta_i}}\right)\left(a - \sum_{j=1}^{K} c_j\right)\right]^{\frac{1}{\gamma}} \tag{10}$$

$$p_k^* = \left(1 - \prod_{i-1} k \frac{n_i}{n_i + \gamma\vartheta_i}\right)\left(a - \sum_{j=1} Kc_j\right) + \sum_{1=1} kc_i \quad \forall k \in 1, ..., K \tag{11}$$

[13]In order to have gains from trade in this market, we naturally require $a > \sum_{j=1}^{K} c_j$. That is, the consumer's willingness to pay for the first unit (a) must exceed the total costs to produce that unit ($\sum_{j=1}^{K} c_j$).

[14]Note that demand is nonnegative and nonincreasing as well for $a \geq 0, b < 0, \text{and} \gamma < 0$, since in that case second-order conditions are always satisfied. Corbett and Karmarkar (2001) develop a multi-layered Cournot model with linear demand.

The competitive benchmark is characterized by a vector of conjectural variation parameters $(\vartheta_1, \vartheta_2, ..., \vartheta_K) \in \times_{k=1}^{K}[0, n_k]$.

Collusion amongst the n_g firms in layer g (with $n_g > 1$) results in an increase in ϑ_g to $\vartheta_g^c \in (\vartheta_g, n_g]$. It follows straightforwardly from equation (11) that such an increase in any ϑ_k decreases the equilibrium quantity. For $n_g > 1$ and $\vartheta_g^c \in (\vartheta_g, n_g]$, we can write the ratio of collusive output to total competitive output r as:

$$r = \frac{Q^g}{Q^*} = \left(\frac{n_g + \gamma\vartheta_g}{n_g + \gamma\vartheta_g^c}\right)^{\frac{1}{\gamma}}$$

Note that, although both Q^* and Q^g depend on market characteristics of every layer, apart from γ, their ratio is a function only of characteristics of the colluding layer. If $\gamma = 1, r \in [\frac{1}{2}, 1)$, with the lower bound corresponding to perfect competition in layer g in the but-for world. An increase (decrease) in γ above (below) 1 decreases (increases) the lower bound value.[15]

Taken together, we can now characterize cartel profits in layer g in terms of the direct purchaser overcharge as:

$$\lambda_g = \frac{\zeta_g + \sigma_g}{\omega_g} - 1 = \frac{\gamma\vartheta_g}{n_g}\frac{(1-r)}{r(1-r^{\gamma})} - 1$$

Note that $\gamma_g = -1$ if prior to collusion layer g was in perfect competition ($\vartheta_g = 0$ or $n_g \to \infty$). In that case, $\sigma_g = 0$ and the total cartel profit equals the overcharge on the direct purchasers. In all other benchmarks, (positive) cartel profits are always smaller than the direct purchaser overcharge.

3.1 Downstream Damages

Downstream from cartel layer g, it follows from equation (12) that equilibrium prices p_k^* (weakly) increase in all layers $k \geq g$, as each subsequent layer passes on part of the price increase it receives from its suppliers to its customers.[16] A full characterization of which player faces what passed-on price increase allows us to consider the direct purchaser overcharge ratios derived above. To begin with, consider:

$$\lambda_{g+1} = \frac{\gamma\vartheta_{g+1}}{n_{g+1} + \gamma\vartheta_{g+1}}\frac{1 - r^{\gamma+1}}{r(1-r^{\gamma})}$$

[15] In particular, $\lim \gamma \to \infty r = 1$ for all parameter values and $\lim \gamma \to r = e^{-1}$ for $\vartheta_g^c = n_g$ and $\vartheta_g = 0$.

[16] For $k \geq g+1$, this pass-on rate R_K can be expressed as:

$$R_k = \frac{\omega_k}{\zeta_k} = \frac{p_k^g - p_k^*}{p_{k-1}^g - p_{k-1}^*} = \frac{n_k}{n_k + \gamma\vartheta_k}$$

Note that, unless in perfect competition ($\vartheta_k = 0$), each layer will absorb some of the price overcharge it receives. The less competitive a layer is, the lower its pass-on fraction.

This expression immediately reveals that the direct purchaser overcharge must generally be a poor estimator for actual direct purchaser harm. In the case of linear demand, for example, we obtain $Q^* \leq 2Q^g$ from equation (11), resulting in $\lambda_{g+1} \in [0, \frac{3}{2}]$. The upper bound is reached when the pre-cartel equilibrium in layer g was perfectly competitive, so that $Q^* = 2Q^g$.[17] The region only slightly changes when demand is non-linear,

The actual antitrust harm of direct purchasers will typically be small when there is strong competition between them, and/or competigion in layer g was weak to begin with. In these cases, the direct purchaser overcharge will significantly overestimate the actual harm that direct purchasers suffer. In particular, if $n_{g+1} \geq 2$ and $\vartheta_{g+1} \leq 1$, in all cases $\lambda_{g+1} \leq 1$.[18] If, on the other hand, layer $g+1$ is governed by a monopolist or a cartel itself (*e.g.*, $\vartheta_{g+1} = n_{g+1}$), the direct purchaser overcharge turns out to be exact. This is so, for example, if direct purchasers have sufficient market power and pre-cartel competition in the colluding layer was strong. Such examples are specific, however.[19] Next, consider the normalized harm to final consumers:[20]

$$\lambda_C = \frac{\gamma}{\gamma+1} \frac{1-r^{\gamma+1}}{r(1-r^\gamma)} \prod_{i=g+1} K \frac{n_i}{n_i + \gamma\vartheta_i}$$

Again, we find for $\gamma = 1$ that $\lambda_c \in [0, \frac{2}{3}]$ with a slightly changed upper bound for non-linear demand. If all intermediate layers downstream from the cartel are sufficiently competitive, the direct purchaser overcharge underestimates actual final consumer harm and $\lambda_C > 1$. If, instead, there is substantial market power in enough of these layers, the direct purchaser overcharge will overestimate consumer harm and $lambda_C < 1$.

Aggregate downstream welfare effects relate to the direct purchaser overcharge as:

$$\lambda_D = \frac{1-r^{\gamma+1}}{r(1-r^\gamma)} \left(1 - \frac{1}{\gamma+1} \prod_{i=g+1}^{K} \frac{n_i}{n_i + \gamma\vartheta_i} \right) \tag{12}$$

Clearly, γ_D decreases with r, a decrease in competition in one of the downstream layers,

[17] This is the principal result in Basso and Ross (2007)

[18] Verboven and van Dijk propose their "discounts" on the direct purchaser overcharge when awarding damages in direct purchaser lawsuits on the claim that $\lambda_{g+1} \leq 1, i.e.$, that the pass-on effect would always outweigh the output effect, $\omega g + 1 \geq \sigma_{g+1}$ and, therefore, that the direct purchaser overcharge overestimates total harm. Note that this not need be true in our more general setting.

[19] Hellwig (2006) ties his argument for limiting standing to sue to direct purchasers to the claim that the direct purchaser overcharge exactly coincides with the actual harm if the direct purchaser layer is monopolized – and thus overestimates the actual harm in all other cases. Verboven and van Dijk (2007) reproduce this result as a special case in their analysis. The marginal price increases in both papers correspond to a marginal increase in ϑ_g in our setting, for which we obtain $\lambda_{g+1} = \frac{2\vartheta_{g+1}}{n_{g+1}+\vartheta_{g+1}}$, which indeed equals 1 for $\vartheta_{g+1} = n_{g+1}$ – and is smaller otherwise.

[20] Note that λ_c can be written as $\lambda_c = \frac{1}{2}\frac{Q^*+Q^g}{Q^g}\mathbb{R}_\mathbb{C}$, where $\mathbb{R}_\mathbb{C} = \prod_{i=g+1}^{K} R_i = \frac{p_k^g - p_K^*}{p_g^\lambda - p_g^*}$ is the part of the price increase due to the cartel that ends up being paid by the final consumers.

and an increase in the number of downstream layers. Nota also that $lambda_D$ is unaffected by changes in the number of upstream layers and their competitiveness.

In the case of linear demand, $\frac{1}{2}\frac{Q^*+Q^g}{Q^g} \leq \lambda_D < \frac{Q^*+Q^g}{Q^g}$. So we find quite intuitively that downstream harm is greater when the cartel reduces output more, which is the case for example when the pre-cartel equilibrium is more competitive. When all intermediate layers are perfectly competitive, every layer fully passes the overcharge which then is eventually borne by the end users, *i.e.*, $\lambda_D = \lambda_C = \frac{\zeta_C+\sigma_C}{\zeta_C} = \frac{1}{2}\frac{Q^*+Q^g}{Q^g} > 1$. This provides the lower bound of the actual harm. Monopolistic competition in the intermediate downstream layers increases actual downstream antitrust harm, with a strict upper bound of $\lambda_D < \frac{Q^*+Q^g}{Q^g}$[21]. Interestingly, we find here the rationale for awarding treble damages according to Section 4 of the Clayton Act.

Proposition 2 *For linear demand, three times the direct purchaser overcharge is the exact upper bound of total downstream harm, i.e.,* $1 < \lambda_D < 3$.

[21]This upper bound is reached with an infinite number of imperfectly competitive layers downstream. This limit case also implies zero equilibrium quantities, $\lim_{K\to\infty} Q^* = \lim_{K\to\infty} Q^g = 0$ – see equation (11)